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Wave Motion in a Radiating Simple Dissociating Gas

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In this note, some theoretical results regarding the coupled effects of relaxation and radiation on the wave propagation in a simple dissociating gas are summarized. The results are derived using the quasi-equilibrium theory of radiation, taking into consideration some details of the physics of the thermal radiation in an oxygen-like gas, consisting of molecules and atoms only (no ionization). A detailed discussion of the theory is given in Ref. 1.

The equations of motion for the inviscid, one-dimensional, time-dependent flow are taken as

Continuity

$$\rho_t + u\rho_x + \rho u_x = 0 \tag{1}$$

Momentum

$$\rho u_t + \rho u u_x + p_x = 0 \tag{2}$$

Energy

$$\rho h_t + \rho u h_x - p_t - u p_x = Q(\rho, \alpha, T) \equiv A_{\text{tot}} - E_{\text{tot}} \tag{3}$$

Rate

$$\alpha_t + u\alpha_x = \theta^{-1}L(p, \rho, \alpha) \tag{4}$$

in which equations the radiation pressure, the contribution to internal energy by radiation, and the effect of photodissociation are neglected. Furthermore, in the equations, ρ, u, p, h, α, and T have their usual meaning ($T \equiv$ translational temperature), Q is the heat input due to emission and absorption of radiation, and the particular form of the right-hand side of Eq. (4) is adopted after the work by Vincenti.[2] With p, ρ, and α chosen as primary state variables, the equations of state are expressed as $T = T(p, \rho, \alpha)$ and $h = h(p, \rho, \alpha)$.

The Q function in Eq. (3) needs some consideration. We assume that the simple dissociating gas is oxygen-like, such that the gas has a strong and continuous absorption in the UV part of the spectrum (Schumann-Runge), and that this spectrum is the most important contributor to the radiation in the gas. Using the quasi-equilibrium theory of radiation,[3] the total rate of emission E_{tot} is consequently taken as

$$E_{\text{tot}} = 4\pi(1 - \alpha)\rho\rho_N^{-1}\int_{\nu_1}^{\nu_2} \kappa_\nu(T)B_\nu(T)d\nu \tag{5}$$

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where $\kappa_\nu(T)$ is the absorption coefficient of the continuum, and $B_\nu(T)$ is the Planck function; the factor $\rho\rho_N^{-1}$ comes from Beer's law (ρ_N is the standard density), the factor $(1 - \alpha)$ is due to the dissociation process in the gas, and ν_1 and ν_2 are the beginning and end of the continuum spectrum, respectively. However, $\kappa_\nu(T)$ varies weakly with frequency at temperatures of interest and can be regarded as independent of ν in the pertinent frequency region. Because of this, and to facilitate a simple calculation of A_{tot} in Eq. (3), a frequency independent absorption coefficient κ of the continuum is defined in terms of a Planck mean

$$\kappa = (1 - \alpha)\rho\rho_N^{-1}\int_{\nu_1}^{\nu_2} \kappa_\nu(T)B_\nu(T)d\nu \Big/ \int_{\nu_1}^{\nu_2} B_\nu(T)d\nu \tag{6}$$

Using this value of κ for the continuum, we define next an optical thickness ξ by

$$\xi = \kappa_0^{-1}\int_0^x \kappa(x')dx' \qquad \nu_1 < \nu < \nu_2 \tag{7}$$

where κ_0 is some fixed value of κ. With the help of this variable, it is possible to express A_{tot} in a simple integral form. When no solid surfaces are present, the combined result for A_{tot} and E_{tot} gives the following expression for Q:

$$Q = 4\pi\kappa\left[\int_{-\infty}^{+\infty} G(T)E_1(\kappa_0|\xi - \xi'|)d\xi' - 2G(T)\right] \tag{8}$$

where

$$2G(T) = \int_{\nu_1}^{\nu_2} B_\nu(T)d\nu$$

and where $E_1(z)$ is a particular form of the integro-exponential function $E_n(z)$ of the general order n (for definition see Ref. 3, p. 253). For a quantitative determination of the radiative properties, a particularly suitable representation of $\kappa_\nu(T)$ for the present purposes has been given by Sulzer and Wieland.[4]

We assume next that acoustic waves are created harmonically at the origin by some unspecified means. The acoustic approximations to the basic equations are obtained, as usual, by linearization, and we write $h = h_0 + h'$, $\kappa = \kappa_0 + \kappa'$, $u = u'$, etc., where the subscript 0 denotes the uniform conditions in the undisturbed gas, and the primes denote small deviations therefrom. The linearized equations can be reduced to a single equation by introducing a potential function Φ, satisfying identically the linearized momentum equation, i.e.,

$$u' = \Phi_x \qquad p' = -\rho_0\Phi_t \tag{9}$$

with the result that

$$\frac{\partial W_S(\Phi)}{\partial t} = 8\,\frac{\kappa_0 a_{f_0}}{Bo} \times \int_{-\infty}^{+\infty} \operatorname{sgn}(x' - x)E_2(\kappa_0|x - x'|)\,\frac{\partial W_T(\Phi)}{\partial x'}\,dx' \tag{10}$$

where

$$W_S = K_S\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2} - \frac{1}{a_{f_0}^2}\frac{\partial^2}{\partial t^2}\right) + \frac{\partial^2}{\partial x^2} - \frac{1}{a_{e_0}^2}\frac{\partial^2}{\partial t^2} \tag{11}$$

$$W_T = K_T\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x'^2} - \frac{1}{c_{f_0}^2}\frac{\partial^2}{\partial t^2}\right) + \frac{\partial^2}{\partial x'^2} - \frac{1}{c_{e_0}^2}\frac{\partial^2}{\partial^2 t} \tag{12}$$

Here a_{f_0} and a_{e_0} are the frozen and equilibrium *isentropic* speeds of sound, respectively, and c_{f_0} and c_{e_0} are the frozen and equilibrium *isothermal* speeds of sound, respectively. Furthermore, K_S and K_T are related to the relaxation time in the gas, such that in an equilibrium flow (infinite reaction rate) K_S and K_T approach zero, whereas in a frozen flow (zero reaction rate) K_S and K_T approach infinity. Another important parameter in Eq. (10) is the Boltzmann number Bo. It is defined as

$$Bo = \bar{c}_{p0}\rho_0 a_{f0}/(\pi/2)(dG/dT)_{T=T_0} \tag{13}$$

where $\bar{c}_{p_0}$ is the equilibrium value of the specific heat at constant pressure, and the expression $(\pi/2)(dG/dT)_{T=T_0}$ is a measure of the radiative heat-transfer rate in the gas. At thermal conditions of interest in the theory of a simple dissociating gas, $Bo \gg 1$. An equation somewhat similar in form to Eq. (10) was derived and investigated previously by Vincenti and Baldwin,[5] who, however, considered a perfect grey gas in their study.

It can be shown that Eq. (10) has an exact solution of the form

$$\Phi = \text{const} \exp(\delta\omega a_{f_0}{}^{-1}x) \exp[i\omega(t + \lambda a_{f_0}{}^{-1}x)] \quad (14)$$

where ω is the frequency of monochromatic waves, and δ and λ are real numbers. Substitution of Eq. (14) into Eq. (10) leads, after splitting into real and imaginary parts, to the following two simultaneous equations for δ and λ (characteristic equations)

$$K_1 = 8\frac{Bu_f}{Bo}\left\{Bu_f \frac{\delta K_4 - \lambda K_2}{\delta^2 + \lambda^2}\log r + Bu_f \frac{\delta K_2 + \lambda K_4}{\delta^2 + \lambda^2}(\beta_1 + \beta_2) - 2K_4\right\} \quad (15a)$$

$$K_3 = 8\frac{Bu_f}{Bo}\left\{-Bu_f \frac{\delta K_4 + \lambda K_4}{\delta^2 + \lambda^2}\log r + Bu_f \frac{\delta K_4 - \lambda K_2}{\delta^2 + \lambda^2}(\beta_1 + \beta_2) + 2K_2\right\} \quad (15b)$$

where

$$r = [(Bu_f + \delta)^2 + \lambda^2]^{1/2}[(Bu_f - \delta)^2 + \lambda^2]^{-1/2}$$

$$\beta_{1,2} = \tan^{-1}[(\delta^2 + \lambda^2 \mp \delta Bu_f)Bu_f{}^{-1}\lambda^{-1}]$$

$$-\pi/2 < \beta_{1,2} < \pi/2$$

$$K_1 = \delta^2 - \lambda^2 + A^2 - 2\delta\lambda a$$

$$K_2 = \delta^2 - \lambda^2 + C^2 - 2\delta\lambda b$$

$$K_3 = a(\delta^2 - \lambda^2 + 1) + 2\delta\lambda$$

$$K_4 = b(\delta^2 - \lambda^2 + B^2) + 2\delta\lambda$$

The parameter $Bu_f = \kappa_0 a_{f0}\omega^{-1}$ is a nondimensional number called the Bueger number, and A, B, C, a, and b stand for

$$A = a_{f0}/a_{e_0} \qquad B = a_{f0}/c_{f_0} \qquad C = a_{f_0}/c_{e0}$$

$$a = \omega K_S \qquad b = \omega K_T$$

The roots δ and λ to Eqs. (15a) and (15b) always occur in pairs so that, if δ_1 and λ_1 are roots, then $-\delta_1$ and $-\lambda_1$ are roots also. In what follows, we study only the positive roots.

1. Equilibrium Waves (K_S, $K_T \to 0$)

The limiting forms of Eqs. (15a) and (15b) for equilibrium waves are obtained by letting K_S, $K_T \to 0$. Then, for any given finite value of $Bo \gg 1$, there are two pairs of roots (δ_1, λ_1) and (δ_2, λ_2). The first pair corresponds to the classical sound wave with a small amount of damping and a slightly altered classical wave speed. The second pair (δ_2, λ_2) is a radiation-induced wave and has no classical counterpart. The first pair (δ_1, λ_1) is referred to as the weak roots, because δ_1 and λ_1 vary weakly with the parameters; the second pair (δ_2, λ_2) is referred to as the strong roots, because δ_2 and λ_2 are strongly varying functions of the parameters.

The weak pair of roots can be obtained by noting that, in the limit as $Bo \to \infty$, the only solutions to the equilibrium characteristic equations are $\lambda_1 = \pm A$ and $\delta_1 = 0$. For a finite $Bo \gg 1$, the following series of developments of λ_1 and δ_1 satisfies these equations:

$$\lambda_1 = A + Bo^{-2}\lambda_1{}^{(1)} + Bo^{-4}\lambda_1{}^{(2)} + \ldots$$

$$\delta_1 = Bo^{-1}\delta_1{}^{(1)} + Bo^{-3}\delta_1{}^{(2)} + \ldots$$

The first-order corrections $\delta_1{}^{(1)}$ and $\lambda_1{}^{(1)}$ can be written as

$$Bo^{-1}\delta_1{}^{(1)} = 8\frac{Bu_e}{Bo_e}A\left[\left(\frac{C}{A}\right)^2 - 1\right]\left(1 - Bu_e\tan^{-1}\frac{1}{Bu_e}\right) \quad (16a)$$

$$Bo^{-2}\lambda_1{}^{(1)} = 4\frac{Bu_e}{Bo_e}(Bo^{-1}\delta_1{}^{(1)})\left\{\left[1 + 3\left(\frac{C}{A}\right)^2\right] \times \left(1 - Bu_e\tan^{-1}\frac{1}{Bu_e}\right) - \left[\left(\frac{C}{A}\right)^2 - 1\right]\frac{2}{1 + Bu_e{}^2}\right\} \quad (16b)$$

where $Bu_e = A^{-1}Bu_f$ and $Bo_e = A^{-1}Bo$, indicating that these solutions depend only on equilibrium values of the parameters, as must be the case. One finds that $\delta_1{}^{(1)}$ has a maximum value of $Bo^{-1}\delta_1{}^{1)} \simeq 1.84\, Bo_e{}^{-1}A[(C/A)^2 - 1]$ for $Bu_e \simeq 0.66$, and that $\delta_1{}^{(1)} \to 0$ for $Bu_e \to 0, \infty$. Using values of κ_0 and a_{e_0} for oxygen, it is found that the maximum of $\delta_1{}^{(1)}$ occurs at relatively low values of ω, actually in the upper part of the audible sound spectrum.

The strong roots of the equilibrium characteristic equations are not so easily obtained. It can be proved, however, that the strong roots only exist for values of $Bu_e > Bu_e{}^*$, where $Bu_e{}^*$ is given by

$$Bu_e{}^* = \frac{j_0 Bo_e}{8\pi} - \frac{8\pi}{j_0{}^3 Bo_e}\left[\left(\frac{C}{A}\right)^2 - 1\right] + \ldots$$

$$j_0 = 1.19969\ldots \quad (17)$$

At $Bu_e = Bu_e{}^*$, $\lambda_2 = 0$, and $\delta_2 = j_0 A Bu_e$. For $Bu_e > Bu_e{}^*$, λ_2 and δ_2 vary strongly with Bu_e.

2. Frozen Waves (K_S, $K_T \to \infty$)

The limiting forms of Eqs. (15a) and (15b) for frozen waves are obtained by letting K_S and K_T approach infinity. Then equations are obtained which are completely similar to the equilibrium characteristic equations. The values of the roots are obtained by replacing (in the equilibrium solutions) Bu_e by Bu_f, Bo_e by Bo_f, A by 1, and C by B. (The parameter $Bo_f = BoK_S/K_T = c_{p0}\rho_0 a_{f0}/(\pi/2)(dG/dT)_{T=T_0}$, where c_{p0} is the frozen value of the specific heat.)

3. Nonequilibrium Waves (K_S, K_T Finite)

To find the weak pair of roots, it is found that the following series development satifies Eqs. (15a) and (15b):

$$\lambda_1 = \lambda_1{}^{(0)} + Bo^{-1}\lambda_1{}^{(1)} + \ldots$$

$$\delta_1 = \delta_1{}^{(0)} + Bo^{-1}\delta_1{}^{(1)} + \ldots$$

wheere $\lambda_1{}^{(0)}$ and $\delta_1{}^{(1)}$ are the solutions for $Bo \to \infty$. The final formulas for $\lambda_1{}^{(1)}$ and $\delta_1{}^{(1)}$ are rather complicated and will not be given here. However, in the limits a, $b \ll 1$ and a, $b \gg 1$ one has the following expansions.

For a, $b \ll 1$:

$$\lambda_1 = A - \frac{a^2}{8A^3}(A^2 - 1)(1 + 3A^2) + 4\frac{Bu_e}{Bo_e}\frac{a}{A^3} \times \left\{-\frac{(C^2 - A^2)(A^2 - 1)}{1 + Bu_e{}^2} + 2\left(1 - Bu_e\tan^{-1}\frac{1}{Bu_e}\right)\left[A^4 + \frac{b}{a}A^2(B^2 - A^2) - C^2\right]\right\} + \ldots \quad (18a)$$

$$\delta_1 = a\frac{A^2 - 1}{2A} + 8\frac{Bu_e}{Bo_e}A\left[\left(\frac{C}{A}\right)^2 - 1\right] \times \left(1 - Bu_e\tan^{-1}\frac{1}{Bu_e}\right) + \ldots \quad (18b)$$

For a, $b \gg 1$:

$$\lambda_1 = 1 + \frac{1}{8a^2}(A^2 + 3)(A^2 - 1) + 4\frac{Bu_f}{Bo_f}\frac{1}{a} \times \left\{-\frac{A^2(B^2 - 1)}{1 + Bu_f{}^2} + 2\left(1 - Bu_f\tan^{-1}\frac{1}{Bu_f}\right)[B^2A^2 - 1 - (a/b)(C^2 - 1)]\right\} + \ldots \quad (18c)$$

$$\delta_1 = \frac{A^2 - 1}{2a} + 8\frac{Bu_f}{Bo_f}(B^2 - 1)\left(1 - Bu_f \tan^{-1}\frac{1}{Bu_f}\right) + \ldots \quad (18d)$$

The nature of the weak roots is clearly shown by Eqs. (18). There are, in each case, two different kinds of terms that give the effect of radiation as well as relaxation on the waves. In δ_1, the last term for slow oscillations (a, $b \ll 1$) is the equilibrium δ_1, and for fast oscillations (a, $b \gg 1$) is the frozen δ_1 [see Eq. (16a)]. It can be shown that the relaxation causes δ to have a maximum value of $\delta_1^{(0)} = 2^{-3/2}(A^2 - 1)(1 + A^2)^{-1/2}$ at $a^2 = (1 + 3A^2)(A^2 + 3)^{-1}$, i.e., at ultrasonic frequencies. Previously we found that the radiation also caused δ_1 to have a maximum. Therefore, there are in general two peaks in the ω spectrum of δ_1, and the relative magnitude of the peaks depends on the thermal state of the gas. Regarding the strong roots, similar results such as those found in the equilibrium case are obtained, but λ_2, δ_2, and $Bu_f{}^*$ are much more complicated functions of the various parameters.

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Experiment in Solar Orientation of Spin Stabilized Satellite

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Early in 1965, Massachusetts Institute of Technology (MIT) Lincoln Laboratory will orbit an experimental satellite designed to test communications techniques and components. The satellite will be spinning at 180 rpm, and, in order to equalize the satellite skin temperatures and increase solar cell efficiency, an attempt will be made to precess the satellite's spin vector perpendicular to the solar vector and maintain it in that position, using a magnetic torquing system that requires no commands from the ground. The advantage of this orientation to a high area thermal-mass ratio satellite with no batteries aboard is a temperature balance more favorable to power conversion. The orientation will be accomplished by controlling the magnitude and direction of the satellite's magnetic moment, which will be aligned with the largest principal moment of inertial ($\cong$1.5 kg-m^2) of the satellite (coincident with the spin axis). This magnetic moment will interact with the earth's geomagnetic field to produce a controlled orientation torque.

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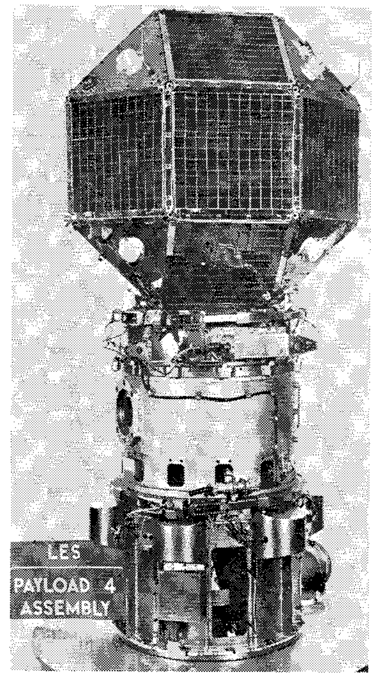

Fig. 1 LES payload 4 assembly.

Construction of a Torquing Device for the Lincoln Experimental Satellite (LES)

The LES is in the form of a 26-sided polyhedron (Fig. 1) with solar cell panels covering all of the square faces. The satellite has a total of eighteen solar panels, eight in an equatorial belt around the spin axis and five above and five below this equatorial belt.

All the solar panels above the satellite equatorial belt and four alternate panels on the belt are connected in parallel. The total output current from these panels is designated I upper. The remaining panels are connected in parallel, and their total current is designated I lower. If these currents are routed in opposite directions in multiturn air-core coils to produce a magnetic moment proportional to their difference, this difference current ΔI can be written in vector form as the dot product of the unit vector along the spin axis $\mathbf{S}$ and the sun vector $\mathbf{SOL}$:

$$\Delta I = I_{\text{upper}} - I_{\text{lower}} = I\mathbf{S}\cdot\mathbf{SOL} \quad (1)$$

The magnetic moment $\mathbf{M}$ produced by this difference current is proportional to the area of the coils A and the number of turns N in each coil and is directed along $\mathbf{S}$:

$$\mathbf{M} = NA\Delta I\mathbf{S} = NAI(\mathbf{S}\cdot\mathbf{SOL})\mathbf{S} \quad (2)$$

Since these coils are in series with the solar panels and the main satellite power bus, it is necessary that the resistance of these coils be low, or all the solar panel current will be turned to heat in these coils, and there will be no power left for the rest of the satellite to function. The present solar cells produce 16 v and 1.4 amp, nominally. In order to limit the coil power consumption to 1% of the total, their voltage drop must not exceed 0.16 v when all 1.4 amp flow through one coil. Thus, the coil resistance is limited to about 0.12 ohm. The total satellite weight is only about 50 lb, and only 1 lb is to be used for the orientation experiment. The area of the coils is limited to about π ft^2 or 0.29 m^2 by the size of the satellite (24 in. across faces). These restrictions on weight, resistance, and coil size restrict the number of turns to about five (with aluminum wire). Hence, the maximum magnetic moment that can be created with an air-core system is $M_{\max}$:

$$M_{\max} = NA\Delta I_{\max} \cong 5(0.29)1.4 \cong 2.0 \text{ amp-m}^2$$